Pinnacle^3® White Paper

P^{3}IMRT

Inverse planning optimization

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Introduction

Intensity-modulated radiation therapy (IMRT) is a 3D planning technique that modulates the intensity of the radiation beams such that a high dose can be delivered to the tumor while significantly sparing the surrounding normal tissues.

IMRT plans are designed using an inverse planning process, which begins by specifying the desired doses to the tumor targets and dose limits to the critical structures. These goals are expressed through a number of weighted dose-volume objectives and/or constraints.

The optimization algorithm divides the beam's eye view of the targets for each beam into a series of finite-sized beamlets. The corresonding weights of the beamlets are optimized to produce a fluence or intensity map for each beam. During optimization, the Delta Pixel Beam™ [1] dose computation is used to determine the dose from the intensity modulated beam. The quality of the plan is scored based on the predefined treatment goals in order to achieve a balance between adequate target coverage and sparing organs at risk.

P^{3}IMRT® is the inverse planning software that is integrated into the Pinnacle^3 treatment planning system. The P^{3}IMRT optimization engine is provided by the ORBIT [2] module RayOptimizer developed by RaySearch Laboratories AB in Stockholm, Sweden.

Objectives and constraints

For tumor targets, the clinical objectives for a P^{3}IMRT optimized plan are expressed in terms of minimum dose (Min Dose), maximum dose (Max Dose), minimum dose to a given volume (Min DVH), maximum dose to a given volume (Max DVH), and uniform dose. Any combination of objectives may be specified. For organs at risk, any combination of maximum dose and maximum dose to a given volume may be used. A weight or penalty factor is assigned to each objective to reflect its importance in the overall treatment objective.

P^{3}IMRT also allows for the use of constraints (objectives that must not be violated) during optimization. Any dose-based objective may be specified as a constraint except the uniform dose objective. In addition, a uniformity constraint can be used to force the dose within the volume to vary by less than a specified percentage.

Typically, multiple objectives and constraints are specified for a given IMRT plan. There may also be multiple objectives or constraints assigned to a single region of interest (ROI). The optimization algorithm attempts to minimize the sum of the n different objectives F^k, k=1...n. This sum is the composite objective function

$$F(\tau) = \sum_{k=1}^{n} F^k$$

where τ is the set of parameters to optimize. Those m functions that are included in the optimization problem as constraints constitute the variable constraints in the problem. These are represented by the constraint vector function

$$C(\tau) \leq 0.$$

Max/Min Dose functions are used if a ROI should receive at most/at least a certain dose level, formulated as

$$F^k = w^k \sum_{i \in V} f\left(d_i, d^k\right) \left(\frac{d_i - d^k}{d^k} \right)^2 \Delta v_i$$

where $f(d_i, d^k)=H(d_i-d^k)$ for the Max Dose function and $f(d_i, d^k)=H(d^k-d_i)$ for the Min Dose function. w^k is the importance weight and H is the Heaviside function. V denotes the ROI in question, d_i the dose in voxel i, and d^k the dose criteria. The voxels' volume is v_i, and Δv_i is the voxel volume relative to the ROI volume. The dose distribution is determined by the optimization parameters, meaning that $d_i = d_i(\tau)$.

The Max/Min DVH functions extend the concept of the Max/Min Dose functions. They are used if a given fraction (η) of the ROI should receive at most/at least the dose d^k.

For Max/Min DVH objectives, the ROI volume is separated into two: the high dose sub-volume V_H and the low dose sub-volume V_L where the separation is determined by the specified fractional volume $1-\eta$ (Max DVH) or η (Min DVH). The Max/Min DVH objectives are evaluated as F^k substituting $V=V_H$ (Max DVH) or $V=V_L$ (Min DVH).

If the dose should be as close as possible to a prescribed level, the Uniform Dose objective is available. This objective is formulated as the Max Dose objective but with $f(d_i, d^k)=1$. This causes the optimization algorithm to attempt to deliver the same dose to every voxel in the ROI. Although this objective will never be fully satisfied, the dose variation will be minimized.

If dose uniformity is important, the Uniformity constraint is available. It flattens the dose distribution in the ROI, thereby minimizing the dose variation. It is implemented as

$$C^k = \sum_{i \in V} \left(\frac{d_i - \bar{d}}{\bar{d}} \right)^2 \Delta v_i$$

where $\bar{d}$ is the mean dose in V. The Uniformity constraint does not specify a dose level, so it should be combined with some other criteria, such as a Uniform Dose objective or a Max/Min DVH function, in order to control the average dose level.

Optimization

RayOptimizer (the P^{3}IMRT optimization engine) utilizes NPSOL [3], a sequential quadratic programming (SQP) algorithm for solving general nonlinear optimization problems. NPSOL employs a dense SQP algorithm [4, 5], and is especially effective for nonlinear problems whose functions and gradients are expensive to evaluate.

The optimization problem that P^{3}IMRT formulates and solves is

$$\min_\tau F(\tau)$$
$$C(\tau) \leq 0,$$
$$\tau \geq 0$$

where F denotes the objective function, C the constraints, and τ the set of parameters to optimize.

The large number of variables and non-negativity bounds presents a large-scale, computation-intensive optimization problem. To improve performance, a reasonable initial estimate $\tau^{(0)}$ of the opening density matrix (ODM) is determined before the actual optimization starts by considering what pixels contribute significantly to the target dose distribution. The ODM is a transmission filter expressed as the relative intensity between the intensity-modulated beam and the open beam exiting the treatment head. Each ODM is discretized over a grid (typically 5 mm resolution). The weight of the corresponding beam elements (pixels) constitutes the optimization variables, which must be non-negative.

Given the initial ODM $\tau^{(0)}$, NPSOL iteratively updates the last found ODM $\tau^{(j)}$ by searching along descent directions so $F(\tau^{(j+1)})<F(\tau^{(j)})$. These descent directions are found by locally approximating the objective function and constraints via quadratics, and by solving corresponding quadratic programming problems.

Typically, the number of variables in IMRT problems is between 1,000 - 10,000. Solving the quadratic programming problem can be very time-consuming. To decrease the computation time, P^{3}IMRT uses an initial solution phase where the bounds are treated in an approximate manner. During this phase much of the computational complexity is avoided. The proper handling of bounds occurs during the remaining iterations, thus assuring an accurate final solution.

References

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[4] P. E. Gill, W. Murray, and M. H. Wright. *Practical Optimization*. New York: Academic Press, 1981.

[5] D. G. Luenberger. *Linear and Nonlinear Programming*. Menlo Park: Addison-Wesley, 1989.

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